## СПИСОК ЭКЗАМЕНАЦИОННЫХ ЗАДАЧ ПО КУРСУ.

### Поток ИП,ВИ 1 курс. Лектор Татаринцев А.В.

### Часть I. Неопределенный интеграл.

*Вычислить интеграл*

1. (a) $\displaystyle\int \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx$ (b) $\displaystyle\int \frac{10^{x-1}-2\cdot 5^{x+1}}{2^x}\,dx$

(c) $\displaystyle\int \frac{x-4}{2x+4\sqrt{x}}\,dx$ (d) $\displaystyle\int \frac{\sqrt{x}-8}{\sqrt{x}-2\sqrt[3]{x}}\,dx$

(e) $\displaystyle\int \frac{1+x^2}{1+x^4}\,dx$ (f) $\displaystyle\int \frac{x^2-1}{\sqrt{x^4+1}}\cdot\frac{dx}{x}$

(g) $\displaystyle\int \frac{dx}{\sin^2 x\cos^2 x}$ (h) $\displaystyle\int \frac{dx}{1+\cos x}$

(i) $\displaystyle\int \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}\,dx$ (j) $\displaystyle\int \frac{dx}{(\sqrt{3}\sin x+\cos x)^2}$

(k) $\displaystyle\int \frac{dx}{\sqrt{x^2-x^4}}$ (l) $\displaystyle\int \frac{x\ dx}{\sqrt{1+x^2+\sqrt{(1+x^2)^3}}}$

(m) $\displaystyle\int \frac{dx}{e^x+e^{-x}}$ (n) $\displaystyle\int \frac{dx}{\sqrt{1+e^{-2x}}}$

(o) $\displaystyle\int \frac{dx}{x\sqrt{3-\ln^2 x}}$ (p) $\displaystyle\int \lg x\cdot\frac{dx}{x}$

(q) $\displaystyle\int \frac{x^3\ dx}{x^8-2}$ (r) $\displaystyle\int x^2\cdot\sqrt[3]{8+x^3}\,dx$

(s) $\displaystyle\int (3x^2-1)\sqrt[3]{x^3-x+2}\,dx$ (t) $\displaystyle\int \frac{(x^2+1)\ dx}{\sqrt[3]{x^3+3x-4}}$

2. (a) $\displaystyle\int \sqrt{\operatorname{arctg}\left(\frac{x}{2}\right)}\cdot\frac{dx}{x^2+4}$ (b) $\displaystyle\int \ln^3 x\cdot\frac{dx}{x}$

(c) $\displaystyle\int e^{\sqrt{x^2+1}}\cdot\frac{x\ dx}{\sqrt{x^2+1}}$ (d) $\displaystyle\int \sin 2x\cdot e^{-3\sin^2 x}\,dx$

(e) $\displaystyle\int \frac{\sin x\ dx}{\sqrt[3]{2+\cos x}}$ (f) $\displaystyle\int \frac{e^x\ dx}{\sqrt{e^{2x}+3}}$

(g) $\displaystyle\int \frac{\cos x\ dx}{3\sin^2 x+4}$ (h) $\displaystyle\int \frac{x^2\ dx}{x^6+2x^3+5}$

(i) $\displaystyle\int \frac{\arcsin^3 x\ dx}{\sqrt{1-x^2}}$ (j) $\displaystyle\int \sin\left(\frac{1}{x^2}\right)\cdot\frac{dx}{x^3}$

(k) $\displaystyle\int \frac{\sin 2x\ dx}{\sqrt{5-\cos^2 x}}$ (l) $\displaystyle\int \frac{dx}{\sqrt{(1-x^2)\sqrt{\arccos x}}}$

(m) $\displaystyle\int \sin\left(\sqrt{x}\right) \cdot \frac{\phantom{dx}}{\sqrt{x}}$ (n) $\displaystyle\int \left(\frac{\phantom{x}}{\sqrt[3]{x}}\right) \, dx$

3. (a) $\displaystyle\int \frac{6x-5}{\sqrt{x^2-4x+13}} \, dx$ (b) $\displaystyle\int \frac{-3x}{x^2+2x+10} \, dx$

(c) $\displaystyle\int \frac{2x+1}{\sqrt{4x-x^2+5}} \, dx$ (d) $\displaystyle\int \frac{3x+4}{x^2+2x+5} \, dx$

(e) $\displaystyle\int \frac{x-3}{\sqrt{4x^2-12x+5}} \, dx$ (f) $\displaystyle\int \frac{10-7x}{x^2+6x+8} \, dx$

(g) $\displaystyle\int \frac{2-7x}{\sqrt{2x-x^2}} \, dx$ (h) $\displaystyle\int \frac{15x-6}{x^2-4x+8} \, dx$

(i) $\displaystyle\int \frac{x+11}{\sqrt{x^2+4x+8}} \, dx$ (j) $\displaystyle\int \frac{5-2x}{\sqrt{2x-x^2+3}} \, dx$

(k) $\displaystyle\int \frac{2x-7}{4x^2-12x+5} \, dx$ (l) $\displaystyle\int \frac{x-2}{4-2x-x^2} \, dx$

(m) $\displaystyle\int \frac{x^2-1}{x^2-2x+5} \, dx$ (n) $\displaystyle\int \frac{(x-2)^2}{x^2+2x+2} \, dx$

4. (a) $\displaystyle\int \left(3-x-x^2\right) \cdot e^{2x} \, dx$ (b) $\displaystyle\int \left(x^2+1\right) \cdot e^{-x} \, dx$

(c) $\displaystyle\int \left(x^2-4x+5\right) \cdot \sin x \, dx$ (d) $\displaystyle\int \left(3x^2+1\right) \cdot \cos 2x \, dx$

(e) $\displaystyle\int x^3 \ln(x+1) \, dx$ (f) $\displaystyle\int \sqrt{x} \ln^2 x \, dx$

(g) $\displaystyle\int \ln\left(x^2+1\right) \, dx$ (h) $\displaystyle\int \ln^3 x \, dx$

(i) $\displaystyle\int x \ln\left(\frac{x+1}{x-1}\right) \, dx$ (j) $\displaystyle\int \left(\frac{\ln x}{x}\right)^2 \, dx$

(k) $\displaystyle\int \frac{2x-1}{\sin^2 x} \, dx$ (l) $\displaystyle\int \frac{x+2}{\cos^2 x} \, dx$

(m) $\displaystyle\int e^{-\sqrt{x}} \, dx$ (n) $\displaystyle\int \cos(\ln x) \, dx$

(o) $\displaystyle\int x \cdot \operatorname{arctg}\left(\frac{x-1}{\sqrt{2}}\right) \, dx$ (p) $\displaystyle\int x \cdot \ln\left(1+\frac{1}{x}\right) dx$

(q) $\displaystyle\int \arcsin^2 x \cdot \frac{x \, dx}{\sqrt{1-x^2}}$ (r) $\displaystyle\int \operatorname{arctg} x \cdot \frac{x \, dx}{\sqrt{x^2+1}}$

(s) $\displaystyle\int \operatorname{arctg} x \cdot x^2 \, dx$ (t) $\displaystyle\int \operatorname{arctg} x \cdot \frac{dx}{x^2}$

(u) $\displaystyle\int \operatorname{arctg} x \cdot \frac{x^2 \, dx}{1+x^2}$ (v) $\displaystyle\int \operatorname{arctg} x \cdot \frac{dx}{x^2\left(1+x^2\right)}$

(w) $\displaystyle\int \arccos\left(\frac{x}{3}\right) \cdot \frac{dx}{\sqrt{x+3}}$ (x) $\displaystyle\int \arcsin\left(\frac{x}{2}\right) \cdot \frac{dx}{\sqrt{x-2}}$

(y) $\displaystyle\int \sin 2x \cdot e^{-5x} \, dx$ (z) $\displaystyle\int \cos 4x \cdot e^{3x} \, dx$

(aa) $\int \sin x \cdot e \; dx$ (ab) $\int x \sin x \cdot e \; dx$

(ac) $\displaystyle\int \cos(\ln x)\ dx$

(ad) $\displaystyle\int \ln(\sin x) \cdot \frac{dx}{\sin^2 x}$

(ae) $\displaystyle\int e^x \cdot \frac{x^2\ dx}{(x+2)^2}$

(af) $\displaystyle\int e^{-x} \cdot \operatorname{arctg} e^x\ dx$

(ag) $\displaystyle\int \ln\left(x + \sqrt{x^2+1}\right) \cdot \frac{x\ dx}{\sqrt{x^2+1}}$

(ah) $\displaystyle\int \ln\left(x + \sqrt{x^2-1}\right) \cdot \frac{dx}{\sqrt{x+1}}$

5. (a) $\displaystyle\int (1 - x\cos x)^2\ dx$

(b) $\displaystyle\int \left(x^2 - \sin x\right)^2\ dx$

(c) $\displaystyle\int (e^x + \cos x)^2\ dx$

(d) $\displaystyle\int \left(e^{-x} - 3x^2\right)^2\ dx$

(e) $\displaystyle\int \left(1 - x - e^{-x}\right) \sin^2 x\ dx$

(f) $\displaystyle\int (1 + 2x\sin x)^2\ dx$

(g) $\displaystyle\int (x - \sin x)\, e^x\ dx$

(h) $\displaystyle\int (1 + x - e^x) \sin x\ dx$

(i) $\displaystyle\int \left(\sqrt{1+x^2} - \cos^2 x\right) x\ dx$

(j) $\displaystyle\int \left[\ln\left(1+x^2\right) - x\sin x\right] x\ dx$

(k) $\displaystyle\int \left(\sqrt{1+x^3} - e^x\right) x^2\ dx$

(l) $\displaystyle\int \left(e^{-x} - \cos x\right) x^2\ dx$

6. (a) $\displaystyle\int x^x(\ln x + 1)\ dx$

(b) $\displaystyle\int x^{-2+1/x}(1 - \ln x)\ dx$

(c) $\displaystyle\int \left(1 - \frac{2}{x}\right)^2 \cdot e^x\ dx$

(d) $\displaystyle\int (\sin x - x\cos x) \cdot \frac{dx}{x^2}$

(e) $\displaystyle\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\ dx$

(f) $\displaystyle\int \left(1 + x - \frac{1}{x}\right) \cdot e^{x+1/x}\ dx$

(g) $\displaystyle\int e^{-\arcsin x}\ dx$

(h) $\displaystyle\int (2x+1)e^{\operatorname{arctg} x}\ dx$

7. (a) $\displaystyle\int \frac{(3x+2)\ dx}{x^2(x-1)}$

(b) $\displaystyle\int \frac{(4x-1)\ dx}{x(x+1)^2}$

(c) $\displaystyle\int \frac{(2x+3)\ dx}{x^3 - x}$

(d) $\displaystyle\int \frac{(3x-4)\ dx}{x^2(x+2)}$

(e) $\displaystyle\int \frac{(3x-1)\ dx}{x(x^2+1)}$

(f) $\displaystyle\int \frac{(2x-1)\ dx}{x^2(x+1)}$

(g) $\displaystyle\int \frac{(3-x)\ dx}{x(x^2-1)}$

(h) $\displaystyle\int \frac{(2x-7)\ dx}{x^3 + x^2}$

(i) $\displaystyle\int \frac{(2-x)\ dx}{x(x^2+4)}$

(j) $\displaystyle\int \frac{(3-2x)\ dx}{x^3 + 1}$

(k) $\displaystyle\int \frac{(2x-7)\ dx}{x(x^2-x-2)}$

(l) $\displaystyle\int \frac{(3-x)\ dx}{(x-1)^2(x+1)}$

8. (a) $\displaystyle\int \frac{dx}{x^3-1}$ (b) $\displaystyle\int \frac{dx}{x^3+1}$

(c) $\displaystyle\int \frac{dx}{x^4-1}$ (d) $\displaystyle\int \frac{dx}{x^4+1}$

(e) $\displaystyle\int \frac{x\ dx}{x^3-1}$ (f) $\displaystyle\int \frac{x\ dx}{x^3+1}$

(g) $\displaystyle\int \frac{x^2\ dx}{x^4+1}$ (h) $\displaystyle\int \frac{x^2\ dx}{x^4-1}$

(i) $\displaystyle\int \frac{x^2\ dx}{x^3+1}$ (j) $\displaystyle\int \frac{x^3\ dx}{x^4-1}$

(k) $\displaystyle\int \frac{dx}{(x^2+1)^3}$ (l) $\displaystyle\int \frac{dx}{(x^2-1)^4}$

(m) $\displaystyle\int \frac{x^2\ dx}{(x^2+1)^3}$ (n) $\displaystyle\int \frac{x^4\ dx}{(x^2-1)^4}$

(o) $\displaystyle\int \frac{x\ dx}{(x^3-1)^3}$ (p) $\displaystyle\int \frac{x^2\ dx}{(x^4-1)^2}$

(q) $\displaystyle\int \frac{x\ dx}{(x^3+1)^3}$ (r) $\displaystyle\int \frac{x^2\ dx}{(x^2+1)^4}$

(s) $\displaystyle\int \frac{x^4\ dx}{(x^2-1)^4}$ (t) $\displaystyle\int \frac{x\ dx}{(x^4+1)^3}$

9. (a) $\displaystyle\int \frac{dx}{x+\sqrt{2x+1}}$ (b) $\displaystyle\int \frac{dx}{\sqrt{x}+\sqrt[4]{x}}$

(c) $\displaystyle\int \frac{dx}{\sqrt[3]{x+1}-\sqrt{x+1}}$ (d) $\displaystyle\int \frac{dx}{x+\sqrt{x-1}-3}$

(e) $\displaystyle\int \frac{\sqrt{1-x}\ dx}{2+\sqrt[3]{1-x}}$ (f) $\displaystyle\int \frac{1-\sqrt[4]{3-x}}{1+\sqrt{3-x}}\ dx$

(g) $\displaystyle\int \frac{3-\sqrt{x-1}}{x+2\sqrt{x-1}}\ dx$ (h) $\displaystyle\int \frac{x\ dx}{1-\sqrt{x-2}}$

(i) $\displaystyle\int \frac{3-x}{1+\sqrt[3]{x+2}}\ dx$ (j) $\displaystyle\int \frac{\sqrt[3]{x}+2}{\sqrt{x}+\sqrt[6]{x}}\ dx$

(k) $\displaystyle\int \frac{\sqrt[3]{x}-3}{2\sqrt[3]{x}+\sqrt{x}}\ dx$ (l) $\displaystyle\int \frac{(x-3)\ dx}{3+\sqrt{x-2}}$

10. (a) $\displaystyle\int \frac{dx}{x-\sqrt{5x+x^2}}$ (b) $\displaystyle\int \frac{dx}{\sqrt{x^2-2x+4}-2}$

(c) $\displaystyle\int \frac{dx}{3+\sqrt{x^2-3x+9}}$ (d) $\displaystyle\int \frac{dx}{x-\sqrt{x^2-3x+2}}$

(e) $\displaystyle\int \frac{dx}{x+\sqrt{x^2+3x-4}}$ (f) $\displaystyle\int \frac{dx}{\sqrt{4x-x^2+1}-1}$

(g) $\displaystyle\int \frac{dx}{\sqrt{4x^2-3x+1}-2x}$ (h) $\displaystyle\int \frac{dx}{2x+\sqrt{4x^2-x+2}}$

(i) $\displaystyle\int \frac{dx}{2-\sqrt{4-x^2-4x}}$ (j) $\displaystyle\int \frac{dx}{1+\sqrt{2x-x^2+1}}$

(k) $\displaystyle\int \frac{\quad}{x-\sqrt{x^2-x+4}}$ (l) $\displaystyle\int \frac{\quad}{x+\sqrt{x^2-3x+1}}$

11. (a) $\displaystyle\int \frac{dx}{(x+1)\sqrt{x^2+1}}$ (b) $\displaystyle\int \frac{dx}{(x-1)\sqrt{x^2+1}}$

(c) $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-x+1}}$ (d) $\displaystyle\int \frac{dx}{x^2\sqrt{2x^2-2x+1}}$

(e) $\displaystyle\int \frac{dx}{(x+1)^2\sqrt{x^2+1}}$ (f) $\displaystyle\int \frac{dx}{x^3\sqrt{x^2-1}}$

12. (a) $\displaystyle\int \frac{(x^3+x)\,dx}{\sqrt{x^2-2}}$ (b) $\displaystyle\int \frac{(2x^2-3)\,dx}{\sqrt{x^2+x+2}}$

(c) $\displaystyle\int \frac{(3x^2-2)\,dx}{\sqrt{1-x^2}}$ (d) $\displaystyle\int \frac{(x^2-2x)\,dx}{\sqrt{x^2-x+2}}$

(e) $\displaystyle\int \frac{(3x^2-1)\,dx}{\sqrt{x^2-x+1}}$ (f) $\displaystyle\int \frac{(x^3-2)\,dx}{\sqrt{x^2+1}}$

13. (a) $\displaystyle\int x^{-3/2}\cdot\left(3x^{1/4}-1\right)^{4/3}\,dx$ (b) $\displaystyle\int \left(2x^4+3x\right)^{1/2}\,dx$

(c) $\displaystyle\int x^{-2/3}\cdot\left(x^{2/3}+2\right)^{1/2}\,dx$ (d) $\displaystyle\int \left(x^3+\sqrt{x}\right)^{1/2}\,dx$

(e) $\displaystyle\int x^{-3/2}\cdot\left(2x^{1/2}+1\right)^{1/3}\,dx$ (f) $\displaystyle\int x^{-1/2}\cdot\left(x^{3/2}+1\right)^{2/3}\,dx$

(g) $\displaystyle\int x^{2/3}\cdot\left(x^{1/3}-3\right)^{3/2}\,dx$ (h) $\displaystyle\int x^{-1/3}\cdot\left(x^{4/3}-1\right)^{1/2}\,dx$

(i) $\displaystyle\int x^{-5/2}\cdot\left(x^{3/2}-1\right)^{2/3}\,dx$ (j) $\displaystyle\int x^{2/3}\cdot\left(3-x^{1/3}\right)^{3/4}\,dx$

(k) $\displaystyle\int x^{1/3}\cdot\left(3x^{2/3}+2\right)^{1/2}\,dx$ (l) $\displaystyle\int x^{1/2}\cdot\left(2x^{3/2}-5\right)^{1/3}\,dx$

14. (a) $\displaystyle\int \sin^2 x\cos 4x\cos 5x\,dx$ (b) $\displaystyle\int \cos^2 x\cos^2 3x\,dx$

(c) $\displaystyle\int \sin^3 x\cos 7x\,dx$ (d) $\displaystyle\int \sin^2 x\sin 5x\sin 6x\,dx$

(e) $\displaystyle\int \sin^2 x\sin^2 4x\,dx$ (f) $\displaystyle\int \cos^2 x\sin 3x\sin 7x\,dx$

(g) $\displaystyle\int \cos x\sin^2 3x\sin 5x\,dx$ (h) $\displaystyle\int \sin x\sin 5x\sin^2 6x\,dx$

(i) $\displaystyle\int \sin^3 x\sin 5x\,dx$ (j) $\displaystyle\int \cos^2 x\sin^2 3x\,dx$

(k) $\displaystyle\int \sin^2 x\sin 2x\cos 3x\,dx$ (l) $\displaystyle\int \sin^2 3x\sin^2 4x\,dx$

15. (a) $\displaystyle\int \frac{dx}{\sin x}$ (b) $\displaystyle\int \frac{dx}{\cos x}$

(c) $\displaystyle\int \frac{\cos^5 x}{\sin^2 x}\,dx$ (d) $\displaystyle\int \frac{\sin^5 x}{\cos^2 x}\,dx$

(e) $\displaystyle\int \frac{}{\cos^4 x}\, dx$ (f) $\displaystyle\int \frac{}{\cos^3 x}\, dx$

(g) $\displaystyle\int \frac{\cos^5 x}{\sin^3 x}\, dx$ (h) $\displaystyle\int \frac{dx}{\sin^3 x\ \cos^3 x}$

(i) $\displaystyle\int \frac{dx}{\sin x\ \cos^5 x}$ (j) $\displaystyle\int \frac{dx}{\sin^3 x\ \cos^2 x}$

(k) $\displaystyle\int \frac{dx}{\sin^2 x\ \cos^3 x}$ (l) $\displaystyle\int \frac{\sin^6 x}{\cos x}\, dx$

(m) $\displaystyle\int \frac{\sin^4 x}{\cos^3 x}\, dx$ (n) $\displaystyle\int \frac{\sin^2 x}{\cos^5 x}\, dx$

(o) $\displaystyle\int \frac{dx}{\sin^2 x \cos^4 x}$ (p) $\displaystyle\int \frac{dx}{\cos^2 x \sin^6 x}$

(q) $\displaystyle\int \frac{dx}{\sin^4 x}$ (r) $\displaystyle\int \frac{dx}{\cos^6 x}$

(s) $\displaystyle\int \mathrm{tg}^2\, x\ dx$ (t) $\displaystyle\int \mathrm{ctg}^2\, x\ dx$

(u) $\displaystyle\int \mathrm{tg}^4\, x\ dx$ (v) $\displaystyle\int \mathrm{ctg}^6\, x\ dx$

16. (a) $\displaystyle\int \frac{\cos x}{2 + \sin x - 3\cos x}\, dx$ (b) $\displaystyle\int \frac{\sin x}{3 - 2\sin x + 3\cos x}\, dx$

(c) $\displaystyle\int \frac{2 + \cos x}{\sin x - 4\cos x}\, dx$ (d) $\displaystyle\int \frac{2\sin x - 1}{1 + 3\sin x - \cos x}\, dx$

(e) $\displaystyle\int \frac{3\sin x + \cos x}{1 + 2\sin x}\, dx$ (f) $\displaystyle\int \frac{\sin x - 3}{1 + 2\sin x + \cos x}\, dx$

(g) $\displaystyle\int \frac{2 + \cos x}{3\sin x - \cos x}\, dx$ (h) $\displaystyle\int \frac{\sin x - \cos x}{1 + \sin x + \cos x}\, dx$

(i) $\displaystyle\int \frac{3\sin x + 2}{3 + 2\sin x}\, dx$ (j) $\displaystyle\int \frac{3 + 2\cos x}{4 + 3\sin x}\, dx$

(k) $\displaystyle\int \frac{\sin x + \cos x}{3\sin x + 2\cos x + 1}\, dx$ (l) $\displaystyle\int \frac{\sin x}{2\cos x - \sin x + 1}\, dx$

17. (a) $\displaystyle\int \frac{3\sin^2 x - 2}{\sin^2 x + 2\sin x\cos x}\, dx$ (b) $\displaystyle\int \frac{3\cos^2 x + 1}{4\cos^2 x - \sin x\cos x}\, dx$

(c) $\displaystyle\int \frac{\sin^2 x - 3\cos^2 x}{\sin^2 x + 2\sin x\cos x}\, dx$ (d) $\displaystyle\int \frac{2\cos^2 x + 1}{2 - 5\sin x\cos x}\, dx$

(e) $\displaystyle\int \frac{\sin^2 x + 1}{\sin^2 x + 4\sin x\cos x}\, dx$ (f) $\displaystyle\int \frac{3\cos^2 x - 2}{\sin^2 x - 3\sin x\cos x}\, dx$

(g) $\displaystyle\int \frac{1 + 3\sin x\cos x}{1 + 2\cos^2 x}\, dx$ (h) $\displaystyle\int \frac{5\cos^2 x + \sin x\cos x}{\sin^2 x + 3\cos^2 x}\, dx$

(i) $\displaystyle\int \frac{2\cos^2 x + 1}{\sin^2 x - 2\cos^2 x}\, dx$ (j) $\displaystyle\int \frac{2\cos^2 x + 3\sin x\cos x}{\sin^2 x - 3\cos^2 x}\, dx$

(k) $\displaystyle\int \frac{2\sin^2 x + 3\sin x\cos x}{4\cos^2 x - \sin^2 x}\, dx$ (l) $\displaystyle\int \frac{3\sin^2 x + 5\cos^2 x}{3\cos^2 x + \sin^2 x}\, dx$

18. (a) $\int x^2\sqrt{4-x^2}\,dx$ (b) $\int x^2\sqrt{x^2-4}\,dx$

(c) $\int x^2\sqrt{x^2+4}\,dx$ (d) $\int \frac{\sqrt{4-x^2}}{x^2}\,dx$

(e) $\int \frac{\sqrt{x^2-4}}{x^2}\,dx$ (f) $\int \frac{\sqrt{x^2+4}}{x^2}\,dx$

(g) $\int \frac{x^2}{\sqrt{4-x^2}}\,dx$ (h) $\int \frac{x^2}{\sqrt{x^2-4}}\,dx$

(i) $\int \frac{x^2}{\sqrt{x^2+4}}\,dx$ (j) $\int \left(1-\frac{1}{x^2}\right)\cdot\frac{dx}{\sqrt{4-x^2}}$

(k) $\int \left(1-\frac{1}{x^2}\right)\cdot\frac{dx}{\sqrt{x^2+4}}$ (l) $\int \left(1-\frac{1}{x^2}\right)\cdot\frac{dx}{\sqrt{x^2-4}}$

19. (a) $\int \sqrt{\operatorname{tg} x}\,dx$ (b) $\int \sqrt{\operatorname{ctg} x}\,dx$

(c) $\int \frac{dx}{\cos x\,\sqrt{\sin x}}$ (d) $\int \frac{dx}{\sin x\,\sqrt[3]{\cos x}}$

(e) $\int \sqrt{1+\sin x}\,dx$ (f) $\int \frac{\sin x\,dx}{\cos x\,\sqrt{1+\sin^2 x}}$

## Часть II. Определенный интеграл.

1. *Используя определение и равномерное разбиение отрезка интегрирования, вычислить интеграл*

(a) $\int\limits_{-3}^{-2} (2x+1)\,dx$ (b) $\int\limits_{2}^{3} (3x-2)\,dx$

(c) $\int\limits_{-1}^{0} x^2\,dx$ (d) $\int\limits_{1}^{2} (4-2x)\,dx$

(e) $\int\limits_{-1}^{0} (4-x^2)\,dx$ (f) $\int\limits_{1}^{2} (3x+1)\,dx$

2. *Вычислить как предел интегральной суммы:*

(a) $\lim\limits_{n\to+\infty} \frac{1^3+2^3+\dots+n^3}{n^4}$

(b) $\lim\limits_{n\to+\infty} \frac{1^2+3^2+\dots+(2n-1)^2}{n^3}$

(c) $\lim\limits_{n\to+\infty} \frac{1^3+3^3+\dots+(2n-1)^3}{n^4}$

(d) $\lim\limits_{n\to+\infty} \left(\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+\dots+\frac{n}{(n+n)^2}\right)$

(e) $\lim\limits_{n\to+\infty} \frac{}{n}\left(\frac{}{n+1}+\frac{}{n+2}+\dots+\frac{}{n+n}\right)$

(f) $\lim\limits_{n\to+\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\dots+\frac{n}{n^2+n^2}\right)$

3. *Вычислить предел, используя правило Лопиталя или теорему о среднем.*

(a) $\lim\limits_{x\to 0}\left(\frac{1}{x^3}\cdot\int\limits_{x}^{\operatorname{tg} x}\cos\left(\sqrt{t}\right)\,dt\right)$

(b) $\lim\limits_{x\to 0}\left(\frac{1}{x^3}\cdot\int\limits_{\sin x}^{x}\cos\left(t^2\right)\,dt\right)$

(c) $\lim\limits_{x\to 0}\left(\frac{1}{x^3}\cdot\int\limits_{\sin x}^{x\cos x}\frac{\sin t}{t}\,dt\right)$

(d) $\lim\limits_{x\to 0}\left(\frac{1}{x^3}\cdot\int\limits_{\sin x}^{\operatorname{tg} x}e^{t^2}\,dt\right)$

(e) $\lim\limits_{x\to 0}\left(\frac{1}{x^4}\cdot\int\limits_{1+x^2}^{e^{x^2}}\frac{\ln(1+t)}{t}\,dt\right)$

(f) $\lim\limits_{x\to 0}\left(\frac{1}{x^2}\int\limits_{0}^{x\sin x}\cos\left(\pi\cos t\right)\,dt\right)$

4. *Используя формулу Ньютона-Лейбница, вычислить интеграл:*

(a) $\int\limits_{1}^{\sqrt{2}}\frac{\sqrt{2-x^2}}{x}\,dx$

(b) $\int\limits_{1}^{e^2}\ln x\,dx$

(c) $\int\limits_{\pi/4}^{\pi/2}\frac{dx}{\sin^4 x}$

(d) $\int\limits_{0}^{\ln 2}\frac{dx}{\sqrt{1+e^{-2x}}}$

(e) $\int\limits_{e^2}^{e^4}\frac{dx}{x\ln x}$

(f) $\int\limits_{0}^{\pi/4}\frac{x\,dx}{\cos^2 x}$

(g) $\int\limits_{0}^{\pi}e^x\cos^2 x\,dx$

(h) $\int\limits_{0}^{3}\arcsin\left(\sqrt{\frac{x}{x+1}}\right)\,dx$

(i) $I_n=\int\limits_{0}^{\pi/2}\sin^n x\,dx$

(j) $I_n=\int\limits_{0}^{1}(1-x^2)^n\,dx$

5. *Найти среднее значение функции* $f(x)$ *на отрезке* $x\in[a;b]$ :

(a) $f(x)=x\sin^2(\pi x);\quad x\in[1;3]$

(b) $f(x)=x\cos^2(\pi x);\quad x\in[-1;1]$

(c) $f(x)=x^2-x\sin(\pi x);\quad x\in[-2;0]$

(d) $f(x)=x^2\sin(\pi x);\quad x\in[0;2]$

(e) $f(x)=(1-x^2)\cos\left(\frac{\pi x}{2}\right);\quad x\in[-1;1]$

(f) $f(x)=(x^2-1)\cos(\pi x);\quad x\in[1;2]$

(a) $\displaystyle\int_0^{2\pi} \frac{\sin x}{x}\; dx$ (b) $\displaystyle\int_0^{\sqrt{2\pi}} \sin\left(x^2\right)\; dx$

(c) $\displaystyle\int_0^{1} \left(e^{-x} - e^{-x^2}\right)\; dx$ (d) $\displaystyle\int_{1/2}^{2} x^2 \ln x\; dx$

7. *Не вычисляя, определить какой из интегралов больше (объяснить):*

(a) $\displaystyle\int_0^{\pi/2} x \sin^2 x\; dx$ или $\displaystyle\int_0^{\pi/2} (\pi/2 - x) \sin^2 x\; dx$

(b) $\displaystyle\int_0^{\pi/2} x \sin^4 x\; dx$ или $\displaystyle\int_0^{\pi/2} (\pi/2 - x) \sin^4 x\; dx$

(c) $\displaystyle\int_0^{\pi/2} x \sin^2 x\; dx$ или $\displaystyle\int_0^{\pi/2} x \cos^2 x\; dx$

(d) $\displaystyle\int_0^{\pi/4} \frac{\sin x}{x}\; dx$ или $\displaystyle\int_0^{\sqrt{\pi}/2} \sin\left(x^2\right)\; dx$

(e) $\displaystyle\int_0^{\pi/4} \sin x\; dx$ или $\displaystyle\int_0^{\sqrt{\pi}/2} \sin\left(x^2\right)\; dx$

8. *Не вычисляя, доказать равенство:*

(a) $\displaystyle\int_{-1}^{1} f\left(x^2\right)\; dx = 2\int_0^{1} f\left(x^2\right)\; dx$ (b) $\displaystyle\int_{-1}^{1} x f\left(x^2\right)\; dx = 0$

(c) $\displaystyle\int_x^{1} \frac{dt}{t^2+1} = \int_1^{1/x} \frac{dt}{t^2+1}$ (d) $\displaystyle\int_0^{\pi/2} f(\sin x)\; dx = \int_0^{\pi/2} f(\cos x)\; dx$

(e) $\displaystyle\int_0^{x} e^{2xt-t^2}\; dt = e^{x^2}\int_0^{x} e^{-t^2}\; dt$ (f) $\displaystyle\int_{1/2}^{2} \frac{x \ln x}{(1+x^2)^2}\; dx = 0$

(g) $\displaystyle\int_0^{\pi} x f(\sin x)\; dx = \frac{\pi}{2}\int_0^{\pi/2} f(\sin x)\; dx$

9. *Вычислить интеграл как функцию параметра a и построить график:*

(a) $\displaystyle I(a) = \int_0^{1} \left|x^2 - a^2\right|\; dx$

(b) $\displaystyle I(a) = \int_{-1}^{a} \operatorname{sgn}(x^3 - x)\; dx$

(c) $I(a) = \int\limits_0 x^2 \cdot \operatorname{sgn}(x-a)\ dx$

(d) $I(a) = \int\limits_0^1 \operatorname{sgn}(x^2 - ax)\ dx$

(e) $I(a) = \int\limits_0^1 x^2 \cdot |x-a|\ dx$

(f) $I(a) = \int\limits_1^a \left[\dfrac{3}{x}\right]\ dx,\ \ a > 1,\ \ [x]$ – целая часть числа $x$.

10. *Геометрические приложения:*

(a) Вычислить длину линии: $\begin{cases} x &= a\ \cos^3 t; \\ y &= a\ \sin^3 t; \end{cases} \quad t \in [0; 2\pi); \quad a > 0.$

(b) Вычислить площадь области: $\rho(\varphi) = a(1 - \sin 2\varphi);\ \ \varphi \in [0; 2\pi)$.

(c) Вычислить длину линии: $y = -\ln\cos x;\ \ x \in [0; \pi/4]$.

(d) Вычислить длину линии: $\begin{cases} x &= a\ (t - \sin t); \\ y &= a\ (1 - \cos t); \end{cases} \quad t \in [0; 2\pi]; \quad a > 0.$

(e) Вычислить площадь: $\rho(\varphi) = a\sin 4\varphi;\ \ \varphi \in [0; 2\pi)$.

(f) Вычислить площадь: $\rho(\varphi) = a\sqrt{\sin 2\varphi}$.

(g) Вычислить площадь поверхности тела вращения: $y = e^{-x};\ \ x \in [0; \ln 2]$.

(h) Вычислить объем $V_{OY}$ тела вращения: $y = \cos x;\ \ x \in [0; \pi/2]$.

(i) Вычислить площадь поверхности тела вращения: $y = \sin x;\ \ x \in [0; \pi]$.

(j) Вычислить объем $V_{OY}$ тела вращения: $x^2 + y^2 = 2x;\ \ x, y \in [0; 1]$.

## Часть II. Несобственный интеграл.

1. *Вычислить несобственный интеграл или доказать его расходимость:*

(a) $\int\limits_0^{1/2} x \ln^2 x\ dx$

(b) $\int\limits_1^2 \dfrac{dx}{x \ln x}$

(c) $\int\limits_0^1 \dfrac{dx}{\sqrt{x}(x+1)}$

(d) $\int\limits_{-1}^1 \dfrac{dx}{\sqrt[3]{x^2 + 2x + 1}}$

(e) $\int\limits_{-1}^1 \dfrac{dx}{\sqrt{(1-x)^3}}$

(f) $\int\limits_0^1 \ln(x+1) \cdot \dfrac{dx}{\sqrt{x^3}}$

(g) $\int\limits_e^{+\infty} \dfrac{dx}{x(\ln x)^{3/2}}$

(h) $\int\limits_1^{+\infty} \dfrac{x\ dx}{x^4 + 1}$

(i) $\int\limits_{1} \left( \frac{\ln x}{x} \right) \, dx$

(j) $\int\limits_{2} \frac{dx}{x \ln^3 x}$.

(k) $\int\limits_{0}^{+\infty} e^{-\sqrt{x}} \, dx$

(l) $\int\limits_{-\infty}^{0} \sin 3x \cdot e^{x} \, dx$

2. *Исследовать несобственный интеграл на сходимость:*

(a) $\int\limits_{0}^{+\infty} \sin\left(x^2\right) \, dx$

(b) $\int\limits_{0}^{+\infty} \frac{\sqrt{x} \cos x}{x+2} \, dx$

(c) $\int\limits_{-\infty}^{0} e^{1/x} \, \frac{dx}{x^3}$

(d) $\int\limits_{0}^{+\infty} \frac{x \ln x}{(1+x^2)^2} \, dx$

(c) $\int\limits_{0}^{+\infty} e^{-x-1/x} \cdot \frac{dx}{x^2}$

(d) $\int\limits_{0}^{+\infty} \frac{\sin^3 x}{x} \, dx$